Recap :- ① Haldane

Band structure + Chern #

finite size edge modes

Bulk $\longleftrightarrow$ Boundary correspondence

Today $\Rightarrow$ • Kane - Mele model
• Haldane model's extension

Basis of Hamiltonian

$$\mathcal{H} = \begin{pmatrix} \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \end{pmatrix}$$

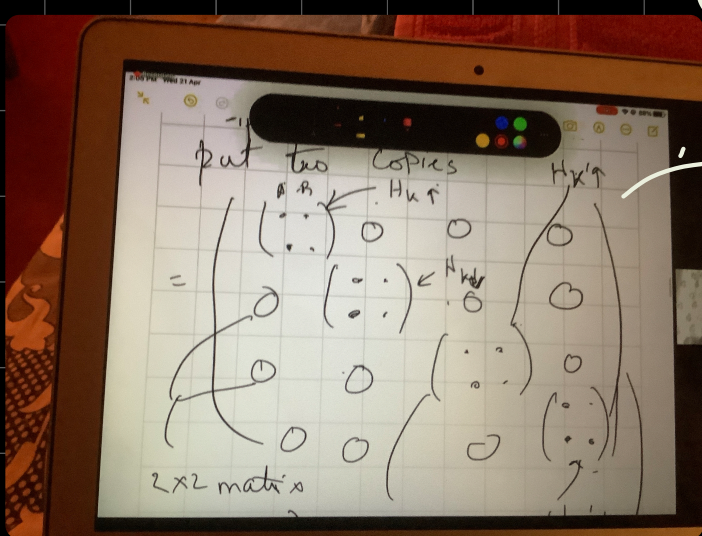
$H_k \begin{pmatrix} a_k \\ b_k \end{pmatrix}$

$H_{k'}$ $\begin{pmatrix} a_{k'} \\ b_{k'} \end{pmatrix}$

valley d.o.f

4x4

Spin dof



spin

8x8 matrix

• Kane & mele → spin & orbit interaction was explicitly added.

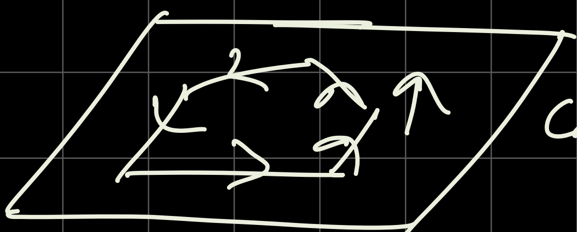
* Haldane model → Chern band without $\vec{B}$

↑ → Chern # +1   ↓ → Chern # (-1)

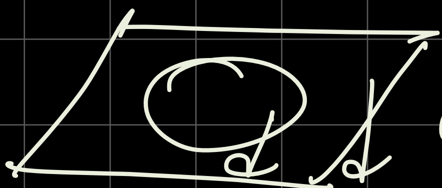
Kane & mele → preserve TRS, no ext $\vec{B}$ field.

↓

invoke spin



Chern# ↑ = +1



Chern # ↓ = -1

Overall Chern # was 0 → but they add up for +1 & -1.

C = 0 → but still top. non trivial

↳ characterizes Z_2 index

$\to \sigma_z$

$$H = t \sum_{\langle ij \rangle} c_i^\dagger c_j + i\lambda_{so} \sum_{\langle\langle ij \rangle\rangle} \nu_{ij}\, c_i^\dagger s^z c_j$$

honeycomb

similar to haldane with $\phi = \frac{\pi}{2}$.

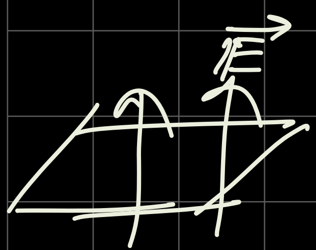
SOC

SOC preserves TRS

$$\vec{L}\cdot\vec{S} \rightarrow (-\vec{L})\cdot(-\vec{S}) = \vec{L}\cdot\vec{S}$$

additional terms

$$+\, i\lambda_{Rashba} \sum_{\langle ij\rangle} c_i^\dagger (\vec{S}\times \hat{\vec{d}}_{ij})_z\, c_j$$



broken mirror symmetry

$$+\, \lambda_v \sum_i \sum c_i^\dagger c_j$$

can break inversion symmetry

breaks sublattice symmetry

• GS ⇒ Topo insulator of 2 Dim.

↳ also called Quantum spin hall syst

#

$$H = \begin{pmatrix} a_k^\dagger & b_k^\dagger \end{pmatrix} \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} a_k \\ b_k \end{pmatrix}$$

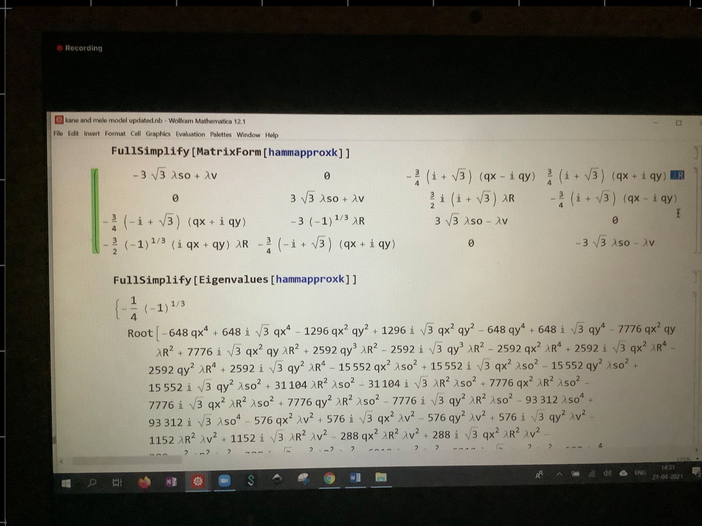
↳ add spins

$$\begin{pmatrix} a_{k\uparrow}^\dagger & a_{k\downarrow}^\dagger & b_{k\uparrow}^\dagger & b_{k\downarrow}^\dagger \end{pmatrix} \begin{pmatrix} (\ ) & \times \\ \times & (\ ) \end{pmatrix} \begin{pmatrix} a_{k\uparrow} \\ a_{k\downarrow} \\ b_{k\uparrow} \\ \vdots \end{pmatrix}$$

b_{k}

SO interaction gives a non-zero int here

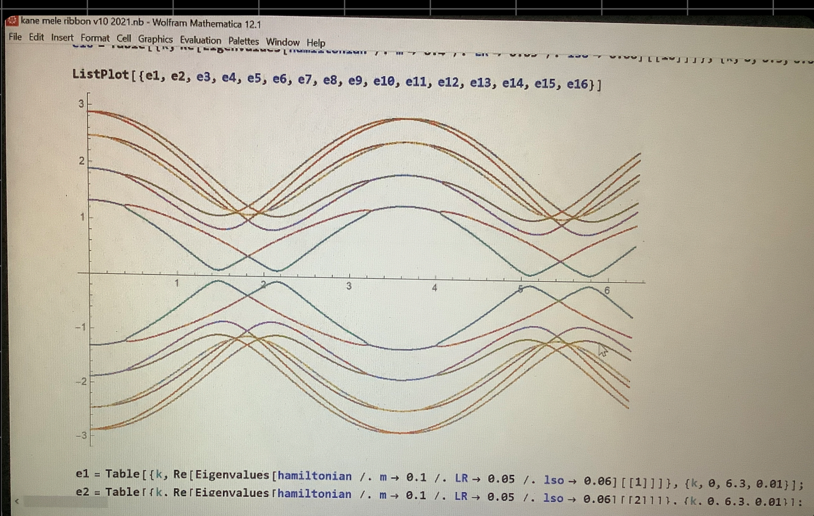
High symmetry points: K, K'



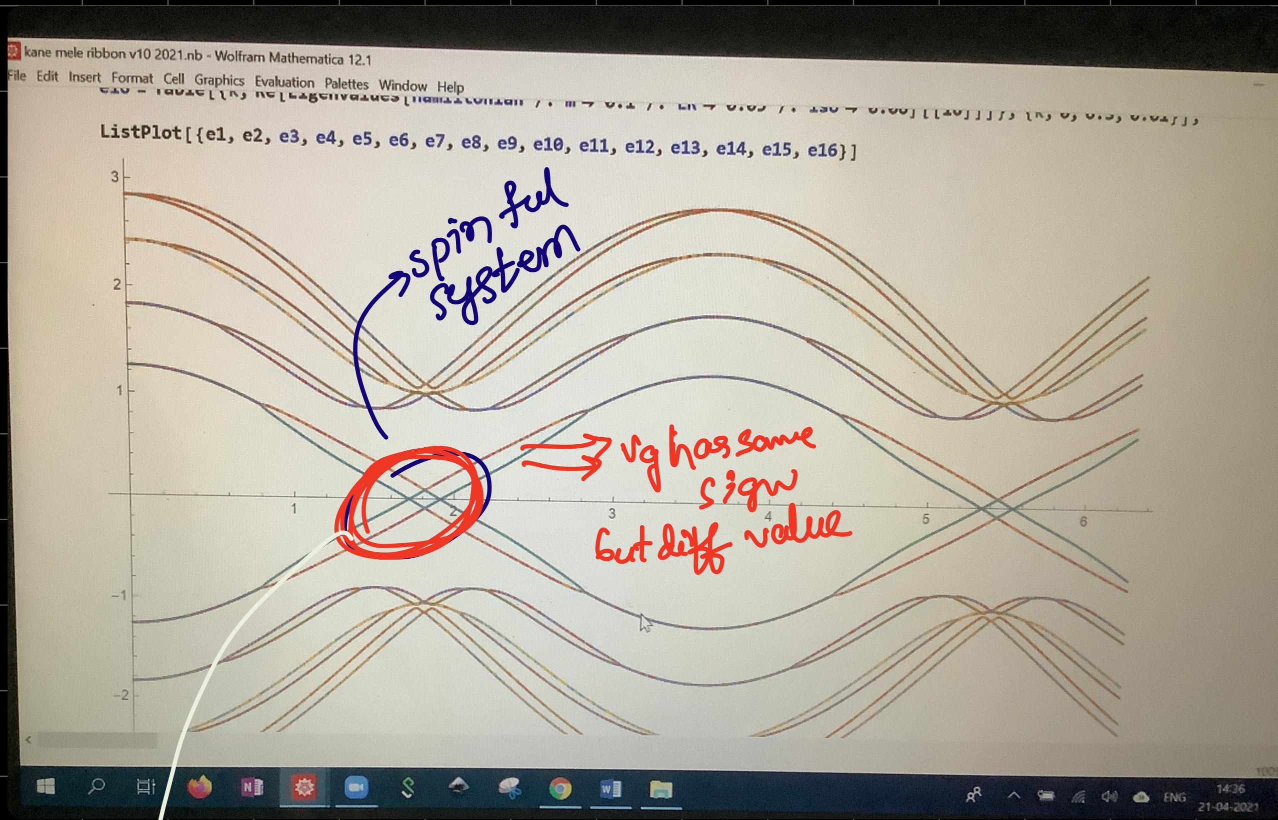
Rashba term ⇒ couples ↑ & ↓ spin

net chern number ⇒ 0 → Spin projected Chern # will be 0.

Finite ribbon calulation



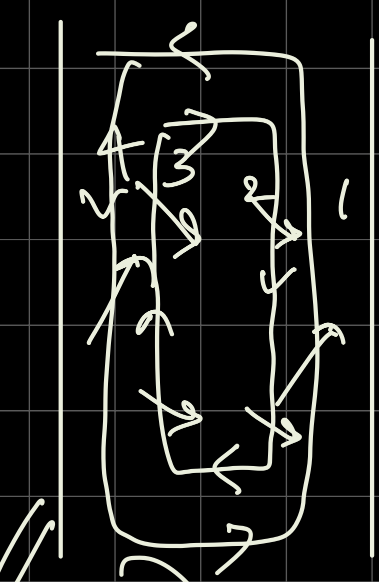
Gapped trivial

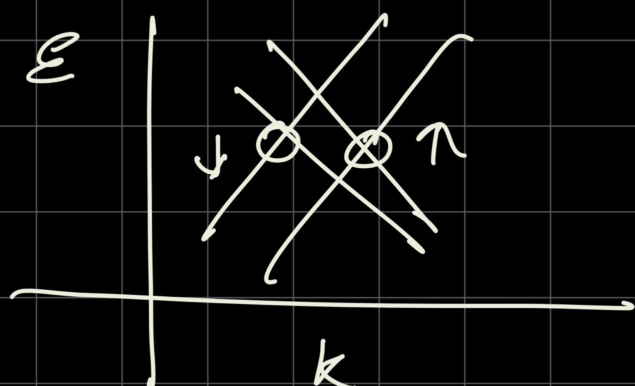


edge modes of the system → have a spin flavour attached to it

(and I'm totally feelin lost here)

→ haldane model

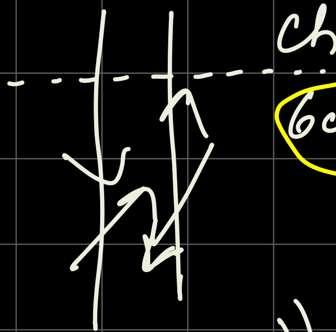




4 states

Quantum spin hall effect → no net charge at the edge

Charge current → 0



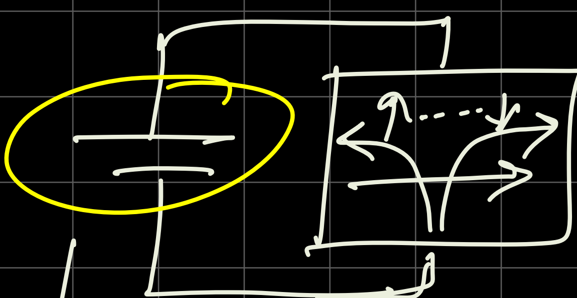
Charge current = 0

but spin current = $\neq 0$ → $i_\uparrow - i_\downarrow$

why QsHall terminology sticks here

But Q. Spin Hall effect

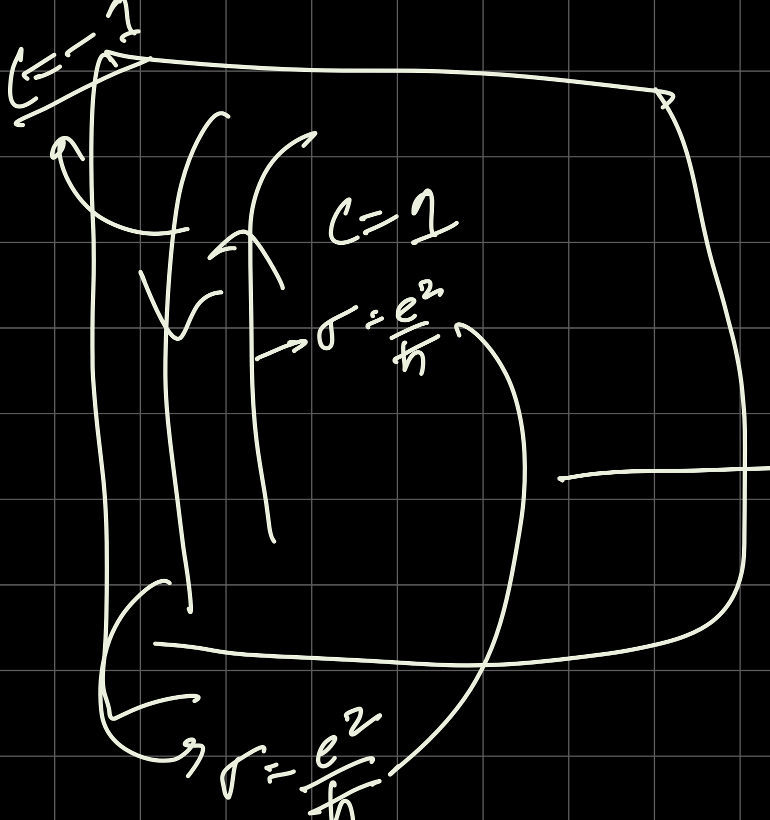
"classical" Spin Hall → material with large atomic #



spin-Hall effect

→ charge current ✓ spin current $\neq 0$

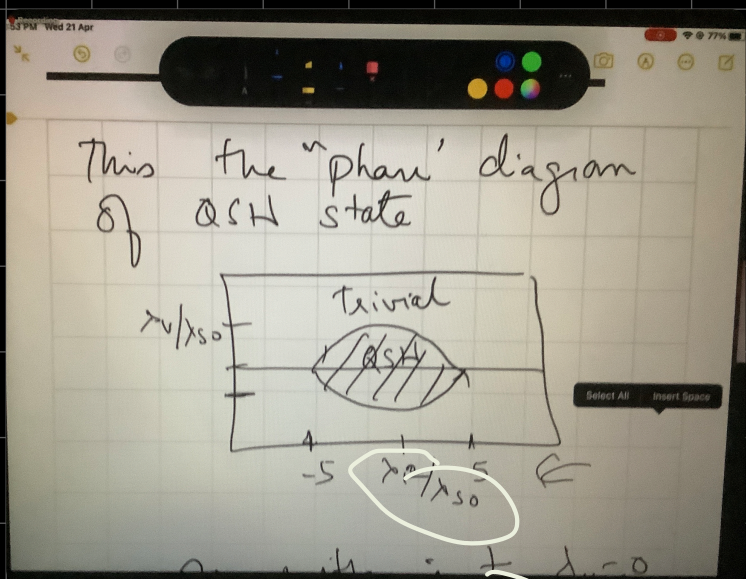
→ non-eqm of spin ↑ more on one side & less on the other side



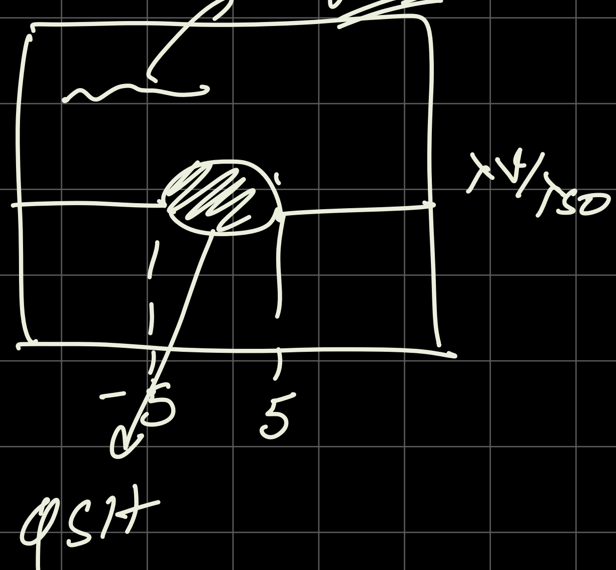
σ_{xy}

→ Chern number.

next class → landauer buttiker picture



gap closing doesn't happen?



mmm → does the job of gap closing

if λ_v is large, then λ_{so} can't close the gap anymore

λ_R / λ_{so} → causes scattering b/w rashba term.

⇒ Shen + Vanderbilt

⇒ Berezovig